Statistics: General Rules of Probability

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This article has covered the introduction to probability and general rules of probability, including the law of large numbers and law of average numbers. The article has also mentioned types of probability i.e. theoretical probability, relative frequency, and subjective probability. The five rules of probability are included in this article, along with probability pitfalls, conditional probability and reversing conditioning.

Definition of Probability

Probability is the **chances of the occurrence of an event** in general. It deals with the study of chances, the study of experiments and their results. The key terms associated with probability are outcomes, events, sample space, and experiment. In layman approach, the probability is the possibility of the happening of a certain event. Probability in statistics is denoted by P and event as E; hence, the probability of an event is denoted by:

P (E)

Formula

The probability of an event can be found by using the following formula:

P (Event) = number of outcomes in the event/total number of outcomes in the sample space

Sample Space is the set of all possible outcomes whereas the **event** is the set of outcomes (possibly empty, possibly all). The main goal of Probability is to use numbers to describe the likelihood of events.

Two assumptions of Probability

- Only finitely many outcomes (i.e., sample space is a finite set)
- All outcomes equally likely

The Law of Large Numbers

The law of large numbers states that if we repeat the same event again and the proportion of occurrence of an event settles at a specific number, this particular number is known as the probability of happening of the event.

Condition

The outcomes of an event should have the same probabilities for each trial.

The Law of Average Numbers

The law of large numbers only refers to long-term events, whereas the average number tells about outcomes related to **short-term events and outcomes**.

Types of Probability

There are three main types of probability:

1. Theoretical probability

For theoretical reasons, we can suppose that all of the n possible outcomes of a particular experiment are equally likely, and we assign a probability of to each possible outcome. For example, if you want to know the theoretical probability that a die will land on a number “3” when rolled, you must determine how many possible outcomes there are. On a die, there are six numbers, offering six possibilities. To land on a three, you have a one-in-six, or 1:6, chance of it landing on a “3”.

2. Relative frequency

Relative frequency is based on observation or actual measurements. Example, when tossing a coin, the total possible outcomes are two, heads and tails. The total number of trials is determined by the total times the coin is flipped. If the coin is flipped 50 times and it lands on heads 28 times, then the theoretical probability is 28/50.

3. Personal or subjective probability

These are values (between 0 and 1 or 0 and 100%) assigned by individuals based on how likely they think events are to occur. For example, during a sport’s game, a fan of one team may state that the team they are rooting for will win. The person bases his decision on facts or opinions regarding the game, the two teams and the likelihood of the team winning.

Five Rules of Probability

There are several rules of probability which should be met in order to define that an event will occur or not, and what is related probability.

Rule 1

If the probability of an event is 0, it indicates that the event will never happen today or in the future. If the probability of an event is 1, it indicates that the event will definitely occur.

Say T is an event which is probable to occur in the near future, and then the probability of occurrence of that event will be denoted as follows:

$$P\ (T)\ 0 \le P\ (A) \le 1$$

Rule 2

A random phenomenon is not very interesting if it has one possible outcome to occur. The probability of the happening of an event can be distributed between the possible outcomes associated with it.

Example: Suppose you are attending the 3^{rd} class of the semester which will continue for a further 33 classes, then there three main possibilities associated with it:

- You will attend the class until the end of the semester.
- You will attend a few more classes in the coming weeks.
- You will not attend any class after the 3^{rd} class.

All three happenings are possible, so the probability associated with all three outcomes is 1. This is known as probability assignment rule and it can be denoted as follows:

$$P(S) = 1$$

Rule 3

The third rule is also known as the complement rule. In case an event as an occurrence probability of 0.30. There are two main possibilities associated with it – the event will occur or it will not occur. The sample space will be made up of both possibilities so, in this case, the probability is that the event will not occur will be shown as follows:

= 1 – P (Event occurs)

$$P (A^{c}) = 1- P (A)$$

Rule 4

This is also known as the addition rule for Disjoint Events. It indicates that if the two events i.e. A and B are disjoint, then the probability of occurrence of these events will be shown as follows:

$$P (A \cup B) = P (A) + P (B)$$

Rule 5

Rule five is based on independent events. It states that if the occurrence of one event does not impact the probability of occurrence of the second one, then both events are independent. The multiplication rule deals with independent events; the multiplication rule is given as follows:

$$P (An \cap B) = P (A) P (B)$$

Probability Pitfalls

- Probabilities which don’t add up to 1 should be avoided.
- If events are not disjoint, then their probabilities should not be added.
- If events are not independent, their probabilities should not be multiplied.
- Disjoint events are dependent events and occurrence of one depends on the

happening of another event.

Conditional Probability

Conditional probability is the **probability of occurrence of an event based on specific events**. Suppose a football match is played between two teams – A and B. Team A has made 4 goals in half an hour, whereas team B has chased the target and made 3 goals within 8 minutes. The rain started right after that and now the probability of Team B winning depends on the stopping of the rain, making the probability of victory conditional.

Conditional probability is not in any way an unnatural or purely theoretical concept. It is completely familiar and natural to you if you have ever bought insurance, played golf, or observed horse racing, to choose just three examples of the myriad available. Thus:

Insurance. Insurance companies require large premiums from young drivers to ensure a car, because they know that P (claim/young driver) > P (claim). For similarly obvious reasons, older customers must pay more for life insurance.

Golf. If you play against the Open Champion then P (you win) ≈ 0. However, given a sufficiently large number of strokes, it can be arranged that P (you win/handicap) ≈ 0.5 Thus any two players can have a roughly even contest.

Horse races. Similarly, any horse race can be made into a much more open contest by requiring faster horses to carry additional weights. Much of the betting industry relies on the judgment of the handicappers in doing this.

The objective of the handicapper in choosing the weights is to equalize the chances to some extent and introduce more uncertainty into the result. The ante-post odds reflect the bookmakers' assessment of how far he has succeeded, and the starting prices reflect the gambler's assessment of the position. Of course, this is not the limit to possible conditions; if it rains heavily before a race then the odds will change to favor horses that run well in heavy conditions. And so on.

Clearly, this idea of conditional probability is relevant in almost any experiment.

Reversing conditioning

Reversing conditioning deals with **inverse probability of an event**. Suppose the probability associated with a person's loss of weight by running 1 hour per day on the treadmill. The reverse probability will be associated with the event that he will not run on the treadmill on a daily basis.

References

Binomial Probability Formula. via mathwords.com

The Binomial Distribution: A Probability Model for a Discrete Outcome.
via sphweb.bumc.bu.edu

Statistics and Probability Dictionary. via stattrek.com

Statistics and Probability Dictionary. via stattrek.com

Probability Models. via stat.yale.edu

Yu, G., 2009. Variance stabilizing transformations of Poisson, binomial and negative binomial distributions. Statistics & Probability Letters, 79(14), pp. 1621-1629.

Notes