Power, Conservative & Nonconservative Forces

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Work is the result that takes place when a force is applied to an object to move it. Work can be measured by multiplying the force by the distance. It is closely related to energy. The concept referred to as work-energy principle states that an increase in the kinetic energy of an object is due to an equal amount of work performed on the object. The unit of work is the joule (J) which equals a newton-meter (N-m).

Work Energy Theorem

W ⇒ work [J], [N * m]

Work is the energy transfer from one object to another so that the second object can be moved or deformed. Like with force, there are several types of work, which are described below.

Work	Equation	Description
Lifting	$W_H = F_G * h$	Lifting an object at constant speed; h = the height that the object is lifted to.
Acceleration Work	$W_B = F_B * s$ $F_B = m * a$	Changing the velocity of an object, making it going faster or slower; s = displacement; F_B= acceleration.
Pressure-Volume Work	$w_V = – \int p\, dV$	The volume of a fluid is compressed (V2 < V1) or expanded (V1 < V2).

Power

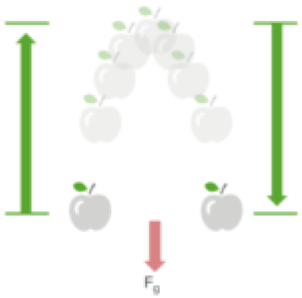
P ⇒ power [W], [J/s]

Power describes the amount of energy consumed per unit of time, i.e., it is dependent on time. The performed work and the incoming energy are inversely proportional to time. That is, with energy remaining constant, power decreases with time.

$$P = \frac{W}{t} = \frac{E}{t}$$

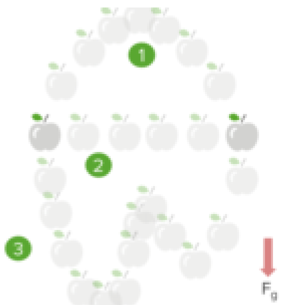
Name	Symbol	Units	Unit abbreviation
Force	F	Newtons = kg m/s^2	N
Energy	K, U, E	Joules = kg m^2/s^2	J
Work	W	Joules = kg m^2/s^2	J
Power	P	Watts = J7s = kg m^2/s^3	W

Conservative Forces

- 

Gravity only converts between K and U.
- Total energy remains constant.
- The net work done by gravity only depends on initial and final location.

Work done by gravity is the same for any path between the same two points.

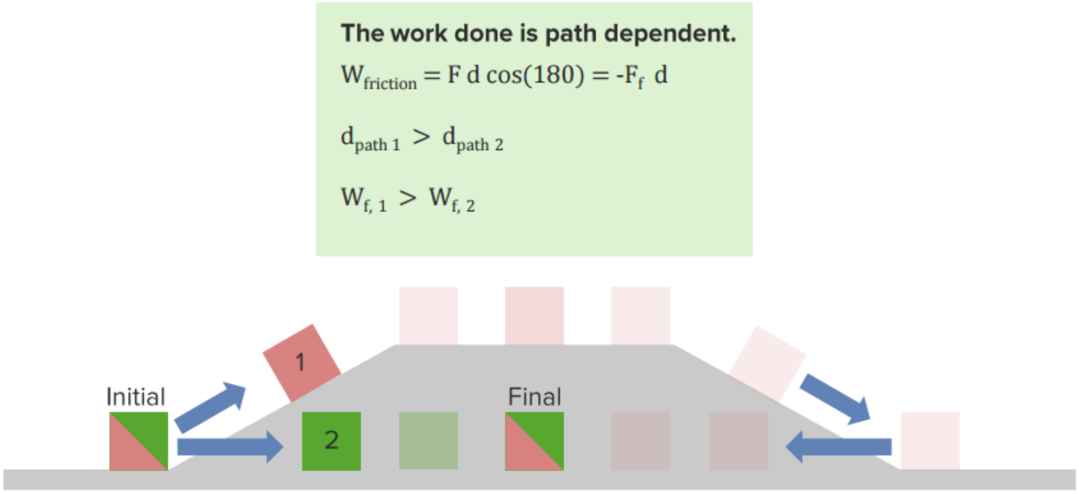
1. 

$W_{Gravity} = 0$ ↑ ↓
2. $W_{Gravity} = 0$ →
3. $W_{Gravity} = 0$ ↓ ↑ $^{\downarrow}$ ↑

The work done by gravity just depends on initial and final locations, regardless of path. Energy is conserved.

Nonconservative Forces

Nonconservative forces are forces that do not store energy and are also known as dissipative forces. A straightforward example is the pushing of a book. When you push a book across a table top, the work that is put into it must be enough to cause the book to move across the table top—but also to combat the friction that occurs in the opposite direction. This type of force, nonconservative force, does not store energy. This is also applicable to air resistance as a plane flies in the air. Any type of friction-type force removes energy from the system and is no longer available to the system for kinetic energy.



Conservative forces	Nonconservative forces
The work done by gravity just depends on initial and final locations, regardless of path.	The work done by friction is path-dependent.
Energy is conserved.	Energy is not conserved (lost to heat).

Dependence on Path

In a nonconservative force, the work that is performed is path-dependent. Work is related to the force that is exerted on the object and the distance the object travels. In other words,
Work = Force x Distance. Since friction is a nonconservative force, the amount of work due to friction will depend on the distance traveled by the object that has a force exerted on it.

For example, if there are two boxes that need to be moved from point A to point B along the same path, then the force that will need to be utilized will be the same for each box. However, let us say Box 1 is moved in a straight line and Box 2 is moved up an incline then down an incline to reach the same destination as Box 1; the force of friction for each box will be different.

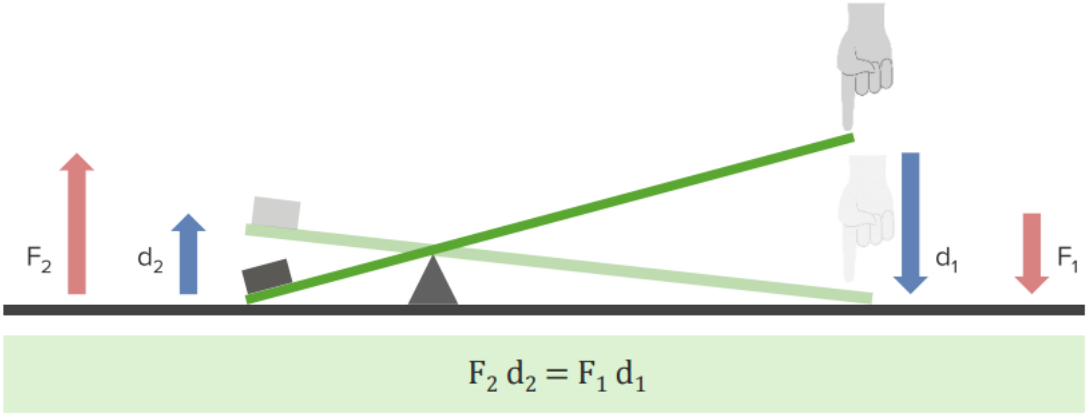
Since Box 2 took a longer path, it will cause the work to move it to be higher since the distance traveled is longer. Thus, the distance for Box 2 is longer than the distance for Box 1 causing the work to move Box 2 to be greater than the work to move Box 1. Work is related to the motion of the object plus the work related to friction. Friction always acts in the opposite direction of motion (180 degrees opposite). It acts against the direction of motion and takes away kinetic energy.

In equation form:

$$W_{friction} = \mathbf{F\ d\ cos\ (180) = -}F\ \mathbf{d}$$

Mechanical Advantage

Mechanical advantage is a measure of the force amplification by using a tool, mechanical device or machine system. An example of a device that provides mechanical advantage is a lever (seesaw) or a pulley system. The device preserves the input power and trades off forces against motion to create the desired amplification.



For a given energy input, trade off distance for force.

Levers

A lever gives the ability to trade off distance for force for a given energy input. In this system, energy is conserved. The force that is applied is used to overcome gravitational energy by pushing (using the force) down on the lever. The lever (seesaw) pivots on a fulcrum. If the fulcrum is placed in the middle of the lever, the force exerted on one side will be equally applied to the other side when lifting the object. In this case, the distance between the fulcrum and the force & the fulcrum and the object would be the same.

However, in most instances, the fulcrum is not in the middle. The object is usually closer to the fulcrum than the location of where the force is applied. In this case, the force applied is less than the force generated on moving the object. In order to conserve energy, the distances would need to be different. The force exerted is at a farther distance than the object's distance from the fulcrum.

Thus, we get:

$$\mathbf{F_2 d_2 = F_1 d_1}$$

Pulleys

A pulley system helps reduce the amount of force that is needed to be applied to move an object. If two pulleys are supporting an object with a separate rope, then the force utilized would go to overcome the downward force of gravity. In this case, each pulley would exert FT such that:

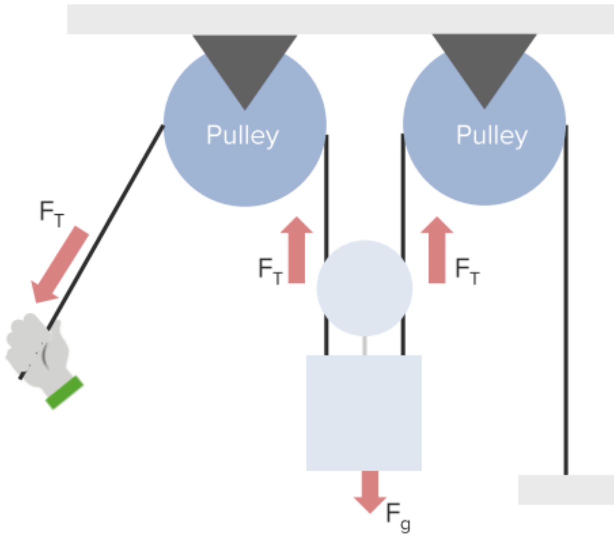
$FT + FT – F_g = 0$ at equilibrium. Thus, $FT = F_g / 2$. This is true since the upward force on the object exerted by the two pulleys must match the downward force of gravity.

Equilibrium

$$F_T + F_T - F_g = m\,a = 0$$

$$2\,F_T = F_g$$

$$F_T = F_g / 2$$



Mechanical Advantage Determination

Mechanical advantage can be determined by looking at the ratio of Force required without using the machine to the Force required using the machine. In other words,

Mechanical advantage = F_{out} / F_{in}.

In the pulley example, it would be F_g / FT = 2 (FT/FT) = 2.

This means that using the pulley system would offer two times mechanical advantage than not using the pulley system.

Now, let us look at an angled ramp. If we have to lift an object straight up, it will require a lot of force to overcome gravitational force without the creation of force of friction. However, if we use a ramp to pull the object up a slope to the height we want the object to be lifted, the force used will be less but will require some force to be used for the force of friction. This is true since a slope provides a gradual force over a distance rather than lifting something straight up.

Mechanical advantage = F_g / F_g sin (theta) = 1 / sin (theta).
In this equation, (theta) is the angle of the slope being utilized. If the (theta) is 30 degrees, then the mechanical advantage using a 30 degree slope = 1 / 0.5 = 2.

Efficiency

Efficiency is associated with how well the force we use is utilized to generate productive work without waste. The key factor that controls efficiency is whether or not friction is generated in the process. Friction is wasted work but sometimes it allows for mechanical advantage.

Friction or No Friction

In the case of pulling an object up a slope, force is used to not only move the object up the slope but also to overcome the frictional force that is opposite the direction of motion. So, more work is utilized to get the object to its final location. However, not all the work is used effectively. Some of the work is used to overcome the frictional force due to the object being dragged across and up the slope. This lost energy is lost in the form of heat. Yet using the slope provides a mechanical advantage over not having used the tool to decrease the force exerted on the object.

$$\text{Efficiency} = \frac{\text{Output work}}{\text{Input work}}$$

No Friction	**Friction**
$\text{Efficiency} = \frac{m\,g\,h}{m\,g\,h}$	$\text{Efficiency} = \frac{m\,g\,h}{m\,g\,h + F_f\,h/\sin(\theta)}$
$\text{Efficiency} = 100\ \%$	$\text{Efficiency} < 100\ \%$

Efficiency is determined by looking at the ratio between Output Work / Input Work.

If there is no friction in the system, efficiency is 100% since all energy in the system is effectively used, and no energy is wasted.

If there is friction in the system, efficiency is < 100% because of loss of energy in the form of heat due to frictional forces. More work is needed to be done, but this does provide a mechanical advantage.

Notes