Force, Uniform Circular Motion & Center of Mass

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Force occurs when two objects interact with each other. Each object exerts a force on the other: a push or a pull. When the interaction between the two objects stops, then the force also disappears. This force can be separated into two categories: forces from contact and forces from action-at-a-distance. The difference between the two classifications is whether the two objects are in direct contact with each other or not. The unit of force is newton (N).

Force and Newton’s Laws

Newton in a 1702 portrait by Godfrey Kneller

First Law: Law of Inertia
An object’s velocity will not change unless acted on by an external net force.

Second Law: The Basic Equation of Mechanics
The sum of forces on an object will be equal to its mass times its acceleration: ***F = m a.***

Third Law: The Law of Action-Reaction
For every force, there is a reaction force equal in magnitude and opposite in direction.

Important Forces

Forces deform objects, set them into motion or accelerate their motion. Force is an interaction between objects that changes the energy of an object. The general equation for force is the following:

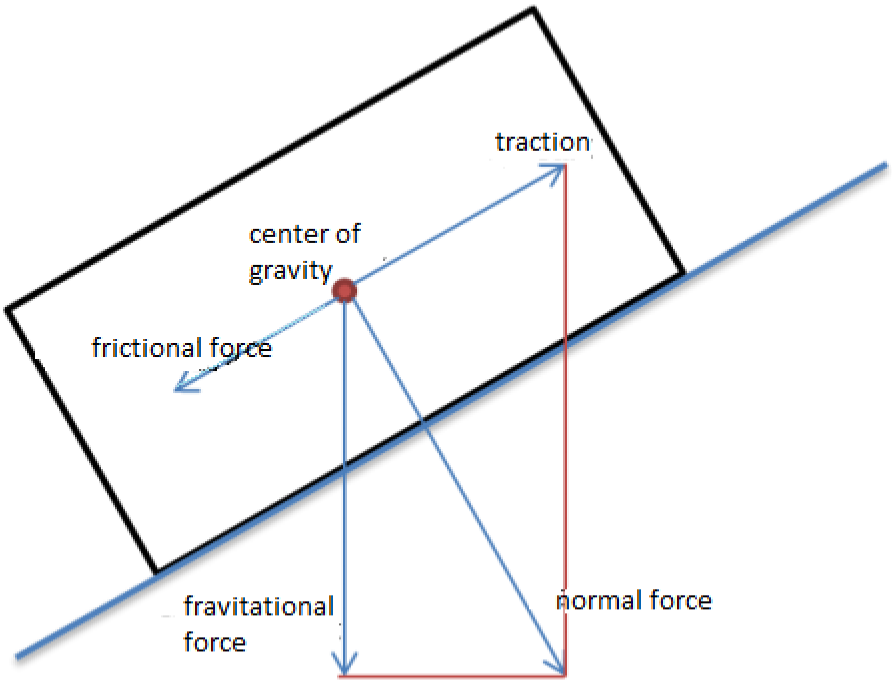
$$F = m * a$$

F ⇒ force [N], [kg * m/s^2]
m ⇒ mass [kg]

Different types of forces are displayed in the equations below:

Force	Equation	Description
Force of Gravity (Weight) F_G	$F_g = m * g$	Force with which the Earth attracts another object towards itself (“downward”). Earth acceleration (gravity) is a constant quantity: g = 9.81 m/s^2.
Buoyancy (Upthrust) F_A	$F_A = \rho_{Fl} * g * V$ $F_A = F_G$ $F_A < F_G$ $F_A > F_G$	Upward force in fluids; $\rho_{Fl\,=}$ density of the fluid [kg/m³]; V = volume of the object placed in the fluid [m³].
Spring Force	$F_D = D * \Delta L$	Deflection of a spring D; Spring constant (material-specific constant) [N/m]; ΔL = the length/stretch of a spring [m].
Normal Force	$F_N = \cos\alpha * F_G$	The force exerted perpendicular to the surface of an object. When exerted horizontally, normal force equals weight. α = angle of the inclined surface measured from the horizontal.
Tangential Force	$F_T = \sin\alpha * F_G$	Force exerted parallel to the surface of an object.
Static Friction Force	$F_H = \mu_H * F_N$	The force exerted by a surface as an object moves across it. It depends on the material and surface quality of the interacting bodies; μ_H = static friction coefficient, without unit.

Kinetic (Sliding) Friction Force	$F_{Gl} = \mu_G * F_N$	The force that occurs when two objects are moving relative to each other and rub together; μ_G = kinetic friction coefficient, without unit.
Rolling Resistance	$F_{Roll} = \mu_{Roll} * F_N$	The force resisting the motion when a body rolls on a surface; μ_{Roll} = rolling resistance coefficient, without unit. Lubricants decrease rolling resistance coefficient, and thus less force is required when moving two colliding bodies.
Centrifugal Force	$F_{Zf} = m * \omega * r$	An inertial force that causes an object in a rotating reference frame to move outward, away from the axis. r = radius of the circle [m]; M = mass of the object ω = angular velocity [1/s].
Centripetal Force	$F_{ZP} = - F_{Zf}$	A force that is directed from the radius toward the center of the circle; opposed to centrifugal force.
Coulomb Force	$F_C = \frac{1}{4\pi * \epsilon_0 \epsilon_R} \frac{Q_1 Q_2}{r^2}$	It describes how strongly two objects or particles are attracted to each other. It depends on the charge of the objects/particles and the distance between them. ε = electric field constant; ε_r = dielectric constant; Q1, Q2 = charge of the two objects [C]; r = distance to the center of the two objects.



Uniform Circular Motion

In this type of motion, an object is moving along a circular path. Since the velocity is a vector, its constantly changing directions balance each other out. Thus, uniform circular motion is defined by the constant sum of velocity. Or to put it simply, if you are driving a car in a circle with the speed of 50 km/h, your acceleration is constant, yet your direction constantly changes.

$$\phi = \frac{1}{2}\alpha * t^2 = \frac{s}{r} = \frac{\omega * t}{2}$$
$$\omega = \frac{v}{r} = \alpha * t = \frac{\pi * n}{30}$$
$$\alpha = \frac{\alpha}{r} = \frac{\omega}{t} = const.$$

ω ⇒ angular velocity [1/s]
α ⇒ angular acceleration [$1/s^2$]
n ⇒ rotational speed [1/s]
r ⇒ radius
π ⇒ Pi (approximated as 3.14)

Center of Mass

The geometric center often differs from the center of mass, since the latter depends on the density (and therefore the mass) of an object. It can be defined as the centroid of a system with any number of points of the same mass A, A_1, A_2… A_n,:

$$M_S = \frac{A_0 + A_1 + A_2 + \cdots + A_n}{n}$$

M_S = Center of mass, no unit.

The entire weight of an object acts at its center of mass (also called centroid). Center of the mass of a human body in a standing posture, for example, lies in the hip area. However, the center of mass can change depending on the posture and motion, and in case of extreme movements, it can even be located outside the body.

The posture of a body determines the type of equilibrium we are in. We distinguish the following types:

- Stable equilibrium: the body comes from the deflected state back to the original posture.
- Unstable equilibrium: after coming back from a deflected state, the body, which had been in equilibrium before, moves further away from the equilibrium state.
- Neutral equilibrium: the body takes a new weight.

Notes