

Force, Uniform Circular Motion & Center of Mass

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Force occurs when two objects interact with each other. Each object exerts a force on the other: a push or a pull. When the interaction between the two objects stops, then the force also disappears. This force can be separated into two categories: forces from contact and forces from action-at-a-distance. The difference between the two classifications is whether the two objects are in direct contact with each other or not. The unit of force is newton (N).



Forces and Newton's Laws of Motion



[Image:](#) Newton in a 1702 portrait by Godfrey Kneller. By Godfrey Kneller, License: Public domain

First law: the law of inertia

An object will remain in either a state of rest or in uniform motion in a straight line unless forces impressed upon it compel the object to change its state. This is the definition of inertia. If there is no net force acting on the object, the object will maintain a constant velocity. If velocity is zero, the object remains at rest.

Second law: the basic equation of mechanics

The vector sum of the forces of an object is equal to mass times acceleration: $F = m a$. In other words, an object's change in momentum is proportional to the impulse placed on the object. It occurs along the straight line upon which the impulse is impressed.

Third law: the law of action-reaction

For every force, there is a reaction force equal in magnitude and opposite in direction. For example, when we walk, we exert a force upon the floor, and the floor exerts a reaction force on us, which causes us to move forward.

Important Forces

Forces deform objects, set them into motion, or accelerate their motion. Force is an interaction between objects that changes the energy of an object. The general equation for force is the following:

$$F = m a$$

$F \Rightarrow$ force (N)

$m \Rightarrow$ mass (kg)

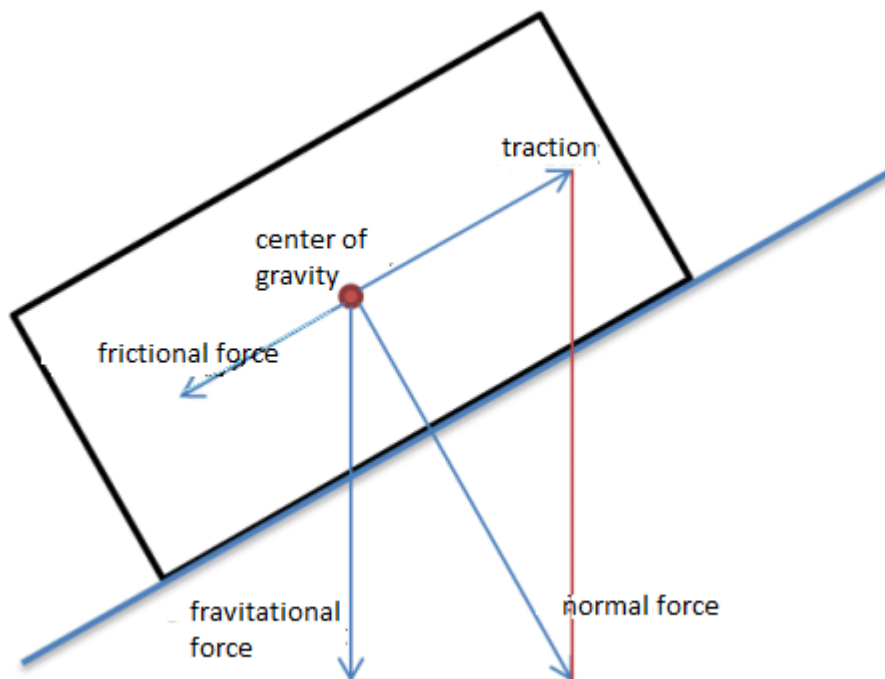
$a \Rightarrow$ acceleration (m/s^2)

Different types of forces are displayed in the equations below:

| Force | Equation | Description |
|---------------------------------|-------------|---|
| Force of gravity (weight) F_G | $F_G = m g$ | F_G is the force with which the earth attracts another object toward itself ('downward'). Earth's acceleration (gravity) is a constant quantity: $g = 9.81 m/s^2$. |

| | | |
|---|---|---|
| Buoyancy (upthrust) F_A | $F_A = \rho_{F1} g V$ $F_A = F_G$ $F_A < F_G$ $F_A > F_G$ | F_A is the upward force in fluids. ρ_{F1} = density of the fluid (kg/m^3) V = volume of the object placed in the fluid (m^3) |
| Spring force F_D | $F_D = D \Delta L$ | F_D describes the deflection of a spring. D = spring constant (material-specific constant) (N/m) ΔL = the length/stretch of a spring (m) |
| Normal force F_N | $F_N = \cos \alpha F_G$ | F_N is the force exerted perpendicular to the surface of an object. When exerted horizontally, normal force equals weight. α = angle of the inclined surface measured from the horizontal |
| Tangential force F_T | $F_T = \sin \alpha F_G$ | F_T is the force exerted parallel to the surface of an object. |
| Static friction force F_H | $F_H = \mu_H F_N$ | F_H is the force exerted by a surface as an object moves across it. It depends on the material and surface quality of the interacting bodies. μ_H = static friction coefficient, without unit |
| Kinetic (sliding) friction Force F_{G1} | $F_{G1} = \mu_G F_N$ | F_{G1} is the force that occurs when 2 objects are moving relative to each other and rub together. μ_G = kinetic friction coefficient, without unit |
| Rolling resistance F_{Roll} | $F_{\text{Roll}} = \mu_{\text{Roll}} F_N$ | F_{Roll} is the force resisting the motion when a body rolls on a surface. μ_{Roll} = rolling resistance coefficient, without unit Lubricants decrease the rolling resistance coefficient, and thus, less force is required when moving 2 colliding bodies. |

| | | |
|----------------------------|--|--|
| Centrifugal force F_{Zf} | $F_{Zf} = m \omega r$ | F_{Zf} is the inertial force that causes an object in a rotating reference frame to move outward, away from the axis. r = radius of the circle (m) m = mass of the object (kg) ω = angular velocity (1/s) |
| Centripetal force F_{Zp} | $F_{Zp} = - F_{Zf}$ | F_{Zp} is a force that is directed from the radius toward the center of the circle. It is opposite of the centrifugal force. |
| Coulomb force F_C | $F_C = \frac{1}{4\pi * \epsilon_0 \epsilon_R} \frac{Q_1 Q_2}{r^2}$ | F_C describes how strongly 2 objects or particles are attracted to each other. It depends on the charge of the objects/particles and the distance between them. ϵ = electric field constant ϵ_r = dielectric constant Q_1, Q_2 = charge of the 2 objects (C) r = distance of the 2 objects to the center (m) |



Uniform Circular Motion

In this type of motion, an object is moving along a circular path. Since the velocity is a vector, its constantly changing directions balance each other out. Thus, uniform circular motion is defined by the constant sum of velocity. For example, if someone drives a car in a circle at 50 m/h, the acceleration is constant, yet the direction constantly changes.

$$\phi = \frac{1}{2} \alpha * t^2 = \frac{s}{r} = \frac{\omega * t}{2}$$

$$\omega = \frac{v}{r} = \alpha * t = \frac{\pi * n}{30}$$

$$\alpha = \frac{a}{r} = \frac{\omega}{t} = const.$$



ω \Rightarrow angular velocity (1/s)

α \Rightarrow angular acceleration (1/s²)

n \Rightarrow rotational speed (1/s)

r \Rightarrow radius (m)

π \Rightarrow Pi (approx. 3.14)

Center of Mass

The geometric center often differs from the center of mass because the latter depends on the density (and therefore, the mass) of an object. It can be defined as the centroid of a system with any number of points of the same mass $A, A_1, A_2 \dots A_n$.

$$M_s = \frac{A_0 + A_1 + A_2 + \dots + A_n}{n}$$

M_s = Center of mass, no unit



Image: Center of gravity. By Lecturio

The entire weight of an object acts at its center of mass (also called centroid). The center of the mass of a human body in a standing posture, for example, lies in the hip area. However, the center of mass can change depending on the posture and motion, and in case of extreme movements, it can even be located outside the body.

The posture of a body determines the type of equilibrium. We distinguish the following types:

- Stable equilibrium: The body comes from the deflected state back to the original posture.
- Unstable equilibrium: After coming back from a deflected state, the body, which had been in equilibrium before, moves further away from the equilibrium state.
- Neutral equilibrium: The body takes a new weight.

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